Tuning gaps and phases of a two-subband system in a quantizing magnetic field

V.M. Apalkov and M.E. Portnoi*
School of Physics, University of Exeter, Stocker Road, Exeter EX4 4QL, United Kingdom

In this work we study the properties of a two-subband quasi-two-dimensional electron system in a strong magnetic field when the electron filling factor is equal to four. When the cyclotron energy is close to the intersubband splitting the system can be mapped onto a four-level electron system with an effective filling factor of two. The ground state is either a ferromagnetic state or a spin-singlet state, depending on the values of the inter-level splitting and Zeeman energy. The boundaries between these phases are strongly influenced by the inter-electron interaction. A significant exchange-mediated enhancement of the excitation gap results in the suppression of the electron-phonon interaction. The rate of absorption of non-equilibrium phonons is calculated as a function of Zeeman energy and inter-subband splitting. The phonon absorption rate has two peaks as a function of intersubband splitting and has a step-like structure as a function of Zeeman energy.

During the last two decades the behavior of a two-dimensional (2D) electron system in a quantizing magnetic field attracted a lot of theoretical and experimental interest. The peculiar properties of this system result from its effective zero dimensionality, because the magnetic field normal to the 2D layer quenches the electron kinetic energy to a constant. As a result the Hamiltonian of the system consists of the interaction part only, and the system undergoes the transition into the incompressible state at certain magnetic field values. The formation of the incompressible states results in the Fractional Quantum Hall Effect - one of the most interesting phenomena discovered in the strongly correlated electron systems.[1–3] The only typical energy of the system is the characteristic electron-electron Coulomb interaction energy, which is of the order of $\varepsilon_C = e^2/\kappa l$, where l is the magnetic length and κ is the dielectric constant. Introduction of a new degree of freedom into this system results into new interesting phenomena when the characteristic energy of the new degree of freedom becomes smaller than the Coulomb energy, ε_C. The electron spin is one of the possible degrees of freedom. If the Zeeman energy is small enough then the transition from spin-polarized to spin-unpolarized ground state occurs at electron filling factor $\nu_0 = 2/m$, where m is odd.[4] At filling factor $\nu_0 = 1/m$ a new type of charged topological excitations, skyrmions, appears.

One can also introduce the degree of freedom in the direction normal to the 2D layer. This can be realized in a double-layer system. In this case the phase diagram is driven by the interplay between the electron-electron Coulomb energy, the tunnelling energy between two layers and the Zeeman energy. It was found that at $\nu_0 = 2$ the double-layer system can be found in three different phases: fully spin-polarized ferromagnetic state, spin-singlet state and canted antiferromagnetic state.[5]

In this paper we consider the system which is similar to a double-layer system but has a different geometry, which provides more experimental possibilities to change the parameters of the system. Namely, we study a single heterojunction in which the second degree of freedom is introduced by the size quantization in the direction perpendicular to the 2D layer. If the electron filling factor is greater than two and the energy $\Delta = \Delta_{12} - \hbar\omega_c$ is smaller than the Coulomb energy, then the second subband has to be taken into account. Here $\hbar\omega_c$ is the cyclotron energy and Δ_{12} is the inter-subband splitting. In this case the second Landau level (Landau level number $n = 1$) of the first subband is close in energy to the first Landau level ($n = 0$) of the second subband. Since the intersubband splitting Δ_{12} has a weak dependence on the magnetic field, the separation Δ can be changed by changing magnetic field B. The influence of the second subband on the optical properties of the magnetically-quantized quasi-2D systems was observed in the different types of experiments, e.g., in extrinsic radiative recombination magnetospectroscopy[6] and in the optically-detected cyclotron resonance in tilted magnetic field.[7] It was proposed in Ref. 8 that the intersection between the levels can be also observed in the magnetic field dependence of the phonon-mediated conductivity of the system. At filling factor slightly greater than two, the dissipative conductivity should reveal a double-peak structure as a function of magnetic filed. A similar double-peak structure should be observed in the other phonon spectroscopy experiments.[9–11] With increasing electron density the repulsion between the levels due to electron-electron exchange interaction opens a gap in the inter-level excitation spectra. As a result the double-peak structure transforms into a single-peak one.[12]

In what follows we study the properties of the two-subband system when the electron filling factor is equal to four, $\nu_0 = 4$. We assume that the Coulomb energy is much smaller than the cyclotron energy. In this case the completely occupied lowest Landau level of the first subband can be considered as a non-dynamical background. We will be interested only in a subsystem consisting of the second Landau level ($n = 1$) of the first subband and the first Landau level ($n = 0$) of the second subband, the spin degeneracy of both levels is lifted by the Zeeman splitting Δ_z. This four-level subsystem is schematically shown in Fig. 1. For $\nu_0 = 4$ the effective filling factor ν of this subsystem is

equal to two, $\nu = 2$. Under this mapping the system becomes similar to a double-layer system with the total filling factor $\nu = 2$ and for its analysis we follow the method of Ref. 5.

For non-interacting electrons we can distinguish three cases: 1) if $\Delta > \Delta_z$ the states '1' and '2' are occupied, and the ground state is a spin singlet state; 2) if $-\Delta_z < \Delta < \Delta_z$ the states '1' and '3' are occupied, and the ground state is a ferromagnetic state; 3) if $\Delta < -\Delta_z$ the states '3' and '4' are occupied, and the ground state is again a spin singlet state. The boundaries between these phases are given by the equations: $\Delta = \Delta_z$ and $\Delta = -\Delta_z$. When the electron-electron interaction is taken into account the transitions between spin singlet and ferromagnetic states can occur through a new phase, for example through a canted antiferromagnetic phase.[5] Our calculations show that in the present system there is no intermediate phase and the transitions from the singlet to the ferromagnetic state are sharp, like in the non-interacting case.

In what follows we use the Coulomb energy ε_C as the unit of energy and magnetic length l as the unit of length. In the Landau gauge with the vector potential $\vec{A} = (0, Bx, 0)$ the eigenstates of a single-electron Hamiltonian are characterized by the Landau level number, n; y-component of the momentum, k_y; the electron subband number, $\mu = 1$ or $\mu = 2$, and the z-projection of the electron spin, $\sigma = 2S_z = \pm 1$:

$$\psi_{n,k_y,\mu,\sigma}(x,y,z) = \chi_\mu(z)\xi_\sigma \frac{e^{ik_y y}}{\sqrt{L_y}}\phi_n(x - k_y) \quad , \tag{1}$$

where $\chi_\mu(z)$ is the envelope wavefunction of the μth subband; ξ_σ is the spin part of the wavefunction; $\phi_n(x)$ is the nth harmonic oscillator function.

The Hamiltonian of the interacting electron system is

$$\begin{aligned} H = & \sum_{n\mu\sigma k_y} \hbar\omega_c \left(n + \frac{1}{2}\right) C^+_{nk_y\mu\sigma} C_{nk_y\mu\sigma} + \frac{\Delta}{2} \sum_{n\mu\sigma k_y} (2\mu - 3) C^+_{nk_y\mu\sigma} C_{nk_y\mu\sigma} + \\ & + \frac{\Delta_z}{2} \sum_{n\mu\sigma k_y} \sigma C^+_{nk_y\mu\sigma} C_{nk_y\mu\sigma} + \frac{1}{2} \sum_{\{n\}} \sum_{\sigma_1\sigma_2} \sum_{\{\mu\}} \sum_{q_x,q_y} \tilde{V}^{(n_1 n_4 n_3 n_2)}_{(\mu_1\mu_4\mu_3\mu_2)}(\hat{q}) \times \\ & \times \sum_{k_1,k_2} e^{iq_x(k_1 - k_2)} C^+_{n_1,k_1+q_y,\mu_1\sigma_1} C^+_{n_2,k_2,\mu_2\sigma_2} C_{n_3,k_2+q_y,\mu_3\sigma_2} C^+_{n_4,k_1,\mu_4\sigma_1} \quad , \end{aligned} \tag{2}$$

where $C^+_{n,k,\mu\sigma}$ and $C_{n,k,\mu\sigma}$ are the creation and annihilation operators of the electron in the state $\psi_{n,k_y,\mu,\sigma}$. In Eq. (2) we use the notations:

$$\tilde{V}^{(n_1 n_4 n_3 n_2)}_{(i_1 i_4 i_3 i_2)}(\hat{q}) = \frac{1}{q} F_{i_1 i_4 i_3 i_2}(q) G_{n_1 n_4}(\hat{q}) G_{n_3 n_2}(\hat{q}^*) \exp\left(-\frac{q^2}{2}\right) \quad , \tag{3}$$

where[13]

$$G_{n_1 n_2}(\hat{q}) = \left(\frac{n_1!}{n_2!}\right)^{1/2} \left(\frac{-i\hat{q}}{\sqrt{2}}\right)^{n_2 - n_1} L^{n_2 - n_1}_{n_1}\left(\frac{q^2}{2}\right) \quad ,$$

$\hat{q} = q_x + iq_y$, $q = |\hat{q}|$ and L^m_n is a generalized Laguerre polynomial. The modification of the Coulomb interaction due to the finite extent of the electron wavefunctions in z-direction is given by:

$$F_{i_1 i_4 i_2 i_3}(q) = \int_0^\infty \int_0^\infty dz_1 dz_2 e^{-q|z_1 - z_2|} \chi_{i_1}(z_1)\chi_{i_4}(z_1)\chi_{i_2}(z_2)\chi_{i_3}(z_2) \quad .$$

We use the Fang-Howard approximation[14] for the envelope wavefunctions of the electrons in the first and the second subbands:

$$\begin{aligned} \chi_1(z) &= \sqrt{\frac{b^3}{2}} z \exp\left(-\frac{1}{2}bz\right) \\ \chi_2(z) &= \sqrt{\frac{b^5}{6}} z \left(z - \frac{3}{b}\right) \exp\left(-\frac{1}{2}bz\right) \quad . \end{aligned}$$

We follow the standard Hartree-Fock method, assuming a non-zero average of $\left\langle C^+_{k_1,\mu_1\sigma_1} C_{k_2,\mu_2\sigma_2} \right\rangle$ over the ground state. The specific feature of the given problem, which differs it from the double-layer system, is that the non-zero

paring in (2) can occur between the states with the different value of k_y, because in our case the two-level system is formed by the different Landau levels.[15] For each value of $k_y = k$ we introduce the new wavefunctions, which are the eigenfunctions of the Hartree-Fock Hamiltonian. The creation and annihilation operators corresponding to the new wavefunctions are $a^+_{k,i}$, $a_{k,i}$, where $i = 1, 2, 3$ or 4. These functions are related to the original ones by the matrix $\gamma_{i,y}$:

$$\begin{array}{l} C_{k,1,-1} = \sum_{i=1}^{4} \gamma_{1i} a_{k,i} \\ C_{k,1,1} = \sum_{i=1}^{4} \gamma_{2i} a_{k,i} \\ C_{k+Q,2,-1} = \sum_{i=1}^{4} \gamma_{3i} a_{k,i} \\ C_{k+Q,2,1} = \sum_{i=1}^{4} \gamma_{4i} a_{k,i} \quad . \end{array} \tag{4}$$

The average of the introduced functions over the ground state is $\left\langle a^+_{k,i} a_{k,j} \right\rangle = \delta_{ij}(\delta_{i1} + \delta_{i2})$. This means that only the states with the lowest energies ($i = 1$ and $i = 2$) are occupied. Substituting Eq.(4) into the Hamiltonian (2) we obtain the Hartree-Fock Hamiltonian in the form of a four-by-four matrix. In the basis $(C_{k,1,-1}, C_{k,1,1}, C_{k+Q,2,-1}, C_{k+Q,2,1})$ the elements of this matrix are

$$\begin{aligned} H^{HF}_{11} &= -\frac{1}{2}(\Delta + \Delta_z) - \beta_{11}\epsilon^{11}_{11} - \beta_{33}\epsilon^{01}_{12} \\ H^{HF}_{22} &= -\frac{1}{2}(\Delta - \Delta_z) - \beta_{22}\epsilon^{11}_{11} - \beta_{44}\epsilon^{01}_{12} \\ H^{HF}_{33} &= \frac{1}{2}(\Delta - \Delta_z) - \beta_{33}\epsilon^{00}_{22} - \beta_{11}\epsilon^{01}_{12} \\ H^{HF}_{44} &= \frac{1}{2}(\Delta + \Delta_z) - \beta_{44}\epsilon^{00}22 - \beta_{44}\epsilon^{01}_{12} \\ H^{HF}_{12} &= -\beta_{12}\epsilon^{11}_{11} - (\beta_{23} - \beta_{14})V^{(0100)}_{(2111)}(Q) - \beta_{34}\epsilon^{01}_{12} \\ H^{HF}_{13} &= -\beta_{13}\left(V^{(0011)}_{(1122)}(0) + V^{(0101)}_{(1212)}(Q)\right) \\ H^{HF}_{14} &= -\beta_{23}V^{(0101)}_{(1212)}(Q) - \beta_{14}V^{(0011)}_{(1122)}(0) \\ H^{HF}_{23} &= -\beta_{14}V^{(0101)}_{(1212)}(Q) - \beta_{23}V^{(0011)}_{(1122)}(0) \\ H^{HF}_{24} &= -\beta_{24}\left(V^{(0011)}_{(1122)}(0) + V^{(0101)}_{(1212)}(Q)\right) \\ H^{HF}_{34} &= -\beta_{34}\epsilon^{00}_{22} - (\beta_{23} - \beta_{14})V^{(0111)}_{(1222)}(Q) - \beta_{12}\epsilon^{01}_{12} \quad , \end{aligned}$$

where $\beta_{ij} = \gamma_{i1}\gamma_{j1} + \gamma_{i2}\gamma_{j2}$,

$$V^{(i_1 i_2 i_3 i_4)}_{(j_1 j_2 j_3 j_4)}(q) = \int_0^\infty dk F_{j_1 j_2 j_3 j_4}(k) \left|G_{i_1 i_2}(k)\right| \left|G_{i_3 i_4}(k)\right| J_{|i_1+i_3-i_2-i_4|}(kq) \exp\left(-\frac{k^2}{2}\right) \quad , \tag{5}$$

$$\epsilon^{i_1 i_2}_{j_1 j_2} = V^{(i_1 i_2 i_1 i_2)}_{(j_1 j_2 j_1 j_2)}(0) \quad ,$$

J_m is the Bessel function of the mth order, and $\epsilon^{i_1 i_2}_{j_1 j_2}$ is the exchange energy of an electron in the i_1th Landau level of the j_1th subband interacting with electrons of the same spin in the filled i_2th Landau level of the j_2th subband.

Taking into account that the eigenvectors of the Hartree-Fock matrix $a_{k,i}$ are related to $C_{k,\mu,\sigma}$ by the matrix γ_{ij} (Eq. (4)), we obtain the self-consistent system of equations for γ_{ij}. The parameter Q can be found by minimizing the Hartree-Fock energy. We have found the solution of the system of equations for γ_{ij} numerically. For all values of Δ and Δ_z the ground state of the system is one of the non-interacting phases (spin singlet or ferromagnetic), described above. The transitions between the phases are sharp. The interaction modifies the phase boundaries only:

$$\Delta = \Delta_z + \frac{1}{2}\left(\epsilon^{00}_{22} - \epsilon^{11}_{11}\right) + \epsilon^{00}_{12} - \epsilon^{01}_{11} + \epsilon^{01}_{12} = \Delta_z + \epsilon_H = \Delta_H \quad , \tag{6}$$

$$\Delta = -\Delta_z + \frac{1}{2}\left(\epsilon^{00}_{22} - \epsilon^{11}_{11}\right) + \epsilon^{00}_{12} - \epsilon^{01}_{11} - \epsilon^{01}_{12} = -\Delta_z + \epsilon_L = \Delta_L \quad , \tag{7}$$

where Δ_H and Δ_L denote the higher and lower values of splitting Δ at the boundary of the ferromagnetic phase, see Fig. 2(a). In Fig. 2(a) the phase diagram is shown for $b = 1$. The phases S_1 and S_2 are the spin-singlet phases

and the phase F is the ferromagnetic phase. The single-electron states '1' & '2', '1' & '3', and '3' & '4' (Fig. 1) are occupied in the phases S_1, F, and S_2, respectively. The electron-electron interaction results in the splitting of the boundaries between different phases, even when the Zeeman energy is equal to zero. For $\Delta_z = 0$ the splitting is equal to $\epsilon_H - \epsilon_L = 2\epsilon_{12}^{01}$, where ϵ_H and ϵ_L are introduced in Eqs. (6) and (7). In Fig. 2(b) the values of the lower (ϵ_L) and upper (ϵ_H) phase boundaries at $\Delta_z = 0$ are shown as the function of the Fang-Howard parameter b, where $1/b$ is proportional to the characteristic width of the heterojunction in units of magnetic length. Both values ϵ_H and ϵ_L decrease with increasing b.

We are mostly interested in the case when the intersubband splitting is close to the cyclotron energy. This gives the restriction of the values of the parameter b. This parameter should be close to the inverse magnetic length. However, the exact relation between b and l for $\Delta = 0$ depends on the actual value of b. Therefore we found it appropriate to calculate the b-dependencies of the main parameters of the system.

The two-subband system can be studied experimentally by changing magnetic field when keeping the fixed value of the filling factor. If the magnetic field $\vec{B}$ is normal to the heterojunction, then both $-\Delta$ and Δ_z increase linearly with increasing B. In this case only the transition from the phase S_1 to the phase F can be observed. Another possibility is to make the experiments in the tilted magnetic field with the fixed value of its normal component. In this case Δ is a constant, because it is proportional to the normal component of the magnetic field, but Δ_z changes linearly with the total magnetic field. Depending on the initial value of Δ, one should observe the transition from S_1 to F or from S_2 to F phases. However, there is a finite region of Δ in which no transitions can be found and the system is always in the phase F. This region has a non-zero range even for zero Δ_z when its width is $2\epsilon_{12}^{01}$.

Now we shall discuss the possibility to detect the different phases by the acoustic-phonon spectroscopy.[9–11] We consider the absorption of non-equilibrium phonons by the discussed quasi-2D system. The electron-phonon interaction is proportional to the electron density operator.[16] During the act of the phonon absorption the electron system undergoes the transition from the ground state to the excited state, corresponding to the density fluctuations. When the system is in the ferromagnetic phase (F), there are spin reversal excitations only. In this case the phonon absorption is forbidden. The collective excitation spectra in the singlet phases S_1 and S_2 can be found from the poles of the density-density correlation function[13], which are given by the expressions:

$$E(q) = \Delta + 2\tilde{V}_{1212}^{0101}(q) - V_{1122}^{0011}(q) - \epsilon_{12}^{01} - \epsilon_{12}^{00} + \epsilon_{11}^{11} + \epsilon_{11}^{01} \tag{8}$$

for the phase S_1 and

$$E(q) = -\Delta + 2\tilde{V}_{1212}^{0101}(q) - V_{1122}^{0011}(q) - \epsilon_{12}^{01} - \epsilon_{11}^{01} + \epsilon_{22}^{00} + \epsilon_{12}^{00} \tag{9}$$

for the phase S_2, where Eqs. (3) and (5) were used. One can rewrite Eqs. (8) and (9) in the form, which can describe both cases simultaneously:

$$E(q) = |\delta\Delta| + \Delta_z + 2\tilde{V}_{1212}^{0101}(q) - V_{1122}^{0011}(q) + \frac{1}{2}\left(\epsilon_{22}^{00} + \epsilon_{11}^{11}\right) \quad , \tag{10}$$

where $\delta\Delta = \Delta - \Delta_H$ for the phase S_1 and $\delta\Delta = -\Delta + \Delta_L$ for the phase S_2.

The energy spectra $E(q)$ are shown in Fig. 3(a) for $\delta\Delta = 0$ and $\Delta_z = 0$, for several different values of parameter b. One can see that there is a finite gap for all values of momentum q. This means that the gap exists for any value of Δ and Δ_z. With increasing the spreading of the electron wavefunction in z direction (decreasing b) the gap becomes smaller, which results from the decreasing of the effective 2D electron-electron interaction. The existence of the interaction-induced finite excitation gap in the two-level system for any value of the inter-level splitting was demonstrated for the system with filling factor $\nu = 1$ in Ref. 12. The effect of the strong renormalization of the excitation energy due to the inter-electron interaction is also known for spin excitations as an interaction enhancement of the g-factor in 2D systems.[18–21]

The electron-phonon interaction Hamiltonian has the form:

$$H_{e-ph} = -\sum_{j,\vec{Q}} \frac{M_j(\vec{Q})}{\sqrt{V}} Z(q_z) \left[\rho(\vec{q})\hat{d}_j^+(\vec{Q}) + \rho(-\vec{q})\hat{d}_j(\vec{Q})\right] \quad , \tag{11}$$

where the isotropic Debye approximation is used with the linear dependence of the phonon frequency on the wave vector: $\omega_j(K) = s_j K$, s_j being speed of sound; j is labeling the phonon modes, $j = 1$ for longitudinal mode and $j = 2, 3$ for two transverse modes. In Eq. (11) we labeled the three dimensional (3D) phonon wave vector by the capital letter, $\vec{Q}$, and its projections by the corresponding small letters, $\vec{Q} = (\vec{q}, q_z)$. The creation operator of a phonon in the jth mode is denoted as $\hat{d}_j^+$, V is a normalization volume, $\rho(\vec{q})$ is the two-dimensional electron density

operator, $M_j(\vec{Q})$ is the matrix element of the electron-phonon interaction, which is determined by the deformation potential and piezoelectric coupling:[17]

$$M_j(\vec{Q}) = \sqrt{\frac{\hbar}{2\rho_0 sQ}}\left[-\beta\frac{Q_xQ_y\xi_{j,z} + Q_yQ_z\xi_{j,x} + Q_zQ_x\xi_{j,y}}{Q^2} - i\Xi_0(\vec{\xi}_j\cdot\vec{Q})\right] \quad , \tag{12}$$

where ρ_0 is the mass density, β and Ξ_0 are the parameters of piezoelectric and deformation potential couplings, $\vec{\xi}_j$ is the polarization vector of the jth phonon mode. The parameters of GaAs are used in our calculations.

Because the phonon-assisted transitions are only allowed between the different subbands, the form factor $Z(q_z)$ is

$$Z(q_z) = \int dz e^{iq_z z}\chi_1(z)\chi_2(z) \quad . \tag{13}$$

At low temperature, $k_BT \ll E(q)$, the rate of absorption of non-equilibrium phonons can be found from the expression:[12,17]

$$\omega_{abs} = \frac{2\pi}{\hbar}\sum_j\int\frac{d\vec{Q}}{(2\pi)^3}\delta(E(q) - s_jQ)n_j(\vec{Q})\left|M_j(\vec{Q})Z(q_z)\right|^2 R_{01}(q) \quad , \tag{14}$$

where $R_{01}(q) = (q^2/2)\exp(-q^2/2)$, $n_j(\vec{Q})$ is the phonon distribution function.

One can see how the appearance of the excitation gap suppresses the phonon absorption. The phonon absorption process requires the conservation of energy and in-plane momentum, $s_j\sqrt{q^2+q_z^2} = E(q)$, where the characteristic excitation gap, $E(q)$, is greater than 0.1 for $b \sim 1$ (see Fig. 3(a)). The GaAs speed of sound in units of $l\epsilon_C/\hbar$ is about 0.03 for longitudinal and 0.02 for transverse phonons. Therefore, the large value of $\sqrt{q^2+q_z^2}$ is required to satisfy the energy conservation law. However, the factors $R_{01}(q)$ and $Z(q_z)$ in Eq. (14) make the phonon absorption to be small at $q > 1$ or $q_z > b$. Thus, the phonon absorption becomes strongly suppressed comparing to the non-interacting case, in which the excitation energy Δ is allowed to have any value. For the non-interacting electrons the absorption rate has two maxima at $|\Delta| \sim \hbar s_j/l$.[8]

From Eq. (14) we calculate the normalized phonon absorption rate, $\omega_0 = \omega_{abs}/n(E(q_0))$, where q_0 is a characteristic momentum, here we use $q_0 = 1$. In Fig. 3(b) the rate ω_0 is shown as a function of Δ for $\Delta_z = 0$ and different values of b. The absorption rate has a two-peak structure. The gap between the peaks corresponds to the phase F of the system, in which the phonons can not be absorbed. The absorption rates for $b = 1$ and $b = 1.5$ are multiplied by 10. With increasing b the collective excitation gap increases (see Fig. 3(a)), which results in a rapid decrease of the absorption rate. One can see this tendency for $b = 0.5$ and $b = 1$. However, with increasing b the phonons with the larger values of q_z can be absorbed, which tends to increase the absorption rate. The latter effect becomes more pronounced for large b when the excitation gap has a weaker dependence on b (see Fig. 3(a)). The competition between these two effects results in the slight increase of the absorption rate for $b = 1.5$.

In the tilted magnetic field experiments, when the Zeeman energy Δ_z is changed while the inter-level splitting Δ is kept fixed, the absorption rate remains constant. For the singlet phase S_1 or S_2 the absorption rate is given by Eqs. (8), (9), (14) and for the ferromagnetic phase F the absorption rate is zero. The dependence of ω_0 on Δ_z is shown schematically in Fig. 3(c), where the critical value of Zeeman splitting, Δ_z^*, corresponds to S_1-F or S_2-F phase transition.

In conclusion, we considered the two-subband electron system with the total filling factor four. The system was mapped onto the four-level electron system with the effective filling factor two. The separation between the levels belonging to the different subbands is proportional to the cyclotron energy and can be changed by changing the magnetic field. The system exists in one of the three different phases: one is ferromagnetic and two are spin-singlet phases. The electron-electron interaction does not create new phases. However, it renormalizes the phase boundaries. The excitation spectrum has the gap for any value of the inter-level splitting Δ and Zeeman splitting Δ_z. This results in the strong suppression of the electron-phonon interaction. The rate of the phonon absorption by the considered quasi-2D electron system has a double-peak structure as a function of level splitting and a step-like structure as a function of Zeeman splitting.

The authors acknowledge the support by the UK EPSRC.

---

* E-mail address: m.e.portnoi@ex.ac.uk

[1] R.E. Prange and S.M. Girvin, *The Quantum Hall Effect* (Springler, Heidelberg, 1987).
[2] T. Chakraborty and P. Pietilainen, *The Fractional Quantum Hall Effect* (Springer, New York, 1995), 2nd ed.
[3] *Perspectives in Quantum Hall Effects*, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).
[4] T. Chakraborty, Adv. Phys. **49**, 959 (2000).
[5] S. Das Sarma, S. Sachdev, and L. Zheng, Phys. Rev. B **58**, 4672 (1998).
[6] A.S. Plaut, I.V. Kukushkin, K.v. Klitzing, and K. Ploog Phys. Rev. B **42**, 5744 (1990).
[7] D.D. Arnone, T.P. Marlow, C.L. Foden, E. H. Linfield, D. A. Ritchie, and M. Pepper, Phys. Rev. B **56**, R4340 (1997).
[8] V.M. Apalkov and M.E. Portnoi, Physica E, to be published; V.N. Golovach and M.E. Portnoi, unpublished.
[9] L.J. Challis and A.J. Kent, in *Aspects of Semiconductor Nanostructures*, edited by G. Bauer, F. Kuchar and H. Heinrich (Springer, Heidelberg, 1992).
[10] C.J. Mellor, R.H. Eyles, J.E. Digby, A.J. Kent, K.A. Benedict, L.J. Challis, M. Henini, C.T. Foxon, and J.J. Harris, Phys. Rev. Lett. **74**, 2339 (1995).
[11] U. Zeitler, A.M. Devitt, J.E. Digby, C.J. Mellor, A.J. Kent, K.A. Benedict, and T. Cheng, Physica B **249-251**, 49 (1998).
[12] V.M. Apalkov and M.E. Portnoi, preprint cond-mat/0111377.
[13] C. Kallin and B.I. Halperin, Phys. Rev. B **30**, 5655 (1984).
[14] T. Ando, A.B. Fowler, and F. Stern, Rev. Mod. Phys. **54** , 437 (1982).
[15] G.F. Giuliani and J.J Quinn, Phys. Rev. B **31**, 6228 (1985).
[16] V.F. Gantmakher and Y.B. Levinson, *Carrier Scattering in Metals and Semiconductors* (North-Holland, Amsterdam, 1987).
[17] K.A. Benedict, R.K. Hills, and C.J. Mellor, Phys. Rev. B **60**, 10984 (1999).
[18] R.J. Nicholas, R.J. Haug, K.v. Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).
[19] B.B. Goldberg, D. Heiman, and A. Pinczuk, Phys. Rev. Lett. **63**, 1102 (1989).
[20] A. Usher, R.J. Nicholas, J.J. Harris, and C.T. Foxon, Phys. Rev. B **41**, 1129 (1990).
[21] A.P. Smith, A.H. MacDonald, and G. Gumbs, Phys. Rev. B **45**, 8829 (1992).

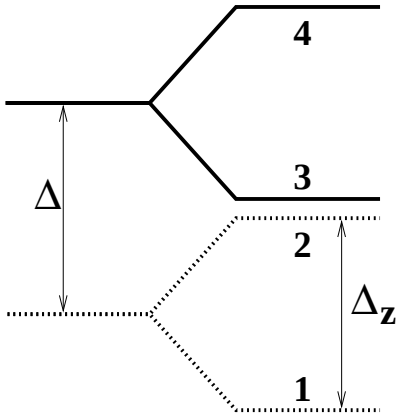

FIG. 1. The single-electron energy levels of the two-subband system. The levels '1' and '2' belong to the second Landau level ($n = 1$) of the first subband with spins $S_z = 1/2$ and $S_z = -1/2$, respectively. The levels '3' and '4' belong to the first Landau level ($n = 0$) of the second subband with spins $S_z = 1/2$ and $S_z = -1/2$, respectively. The splitting Δ is the interlevel splitting, Δ_z is the Zeeman energy.

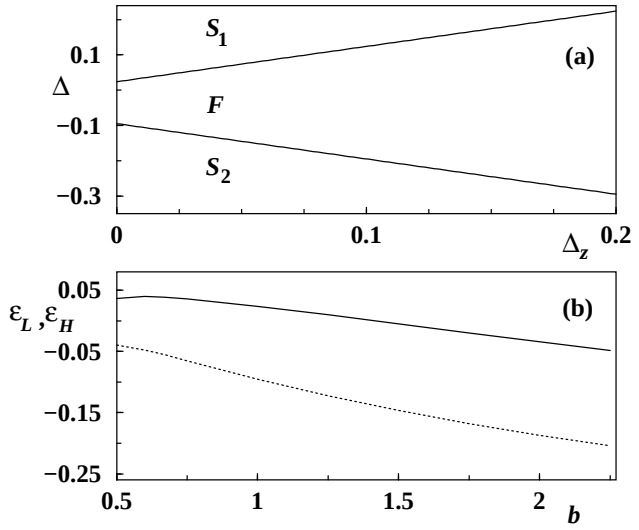

FIG. 2. (a) The phase diagram of the four-level system for $b = 1$. The phases S_1 and S_2 are spin singlet phases, the phase F is a ferromagnetic phase. (b) The parameters ϵ_H and ϵ_L (Eqs.(6)-(7)) of the phase diagram are plotted as the functions of b. All energies are in units of ε_C, b is in units of $1/l$.

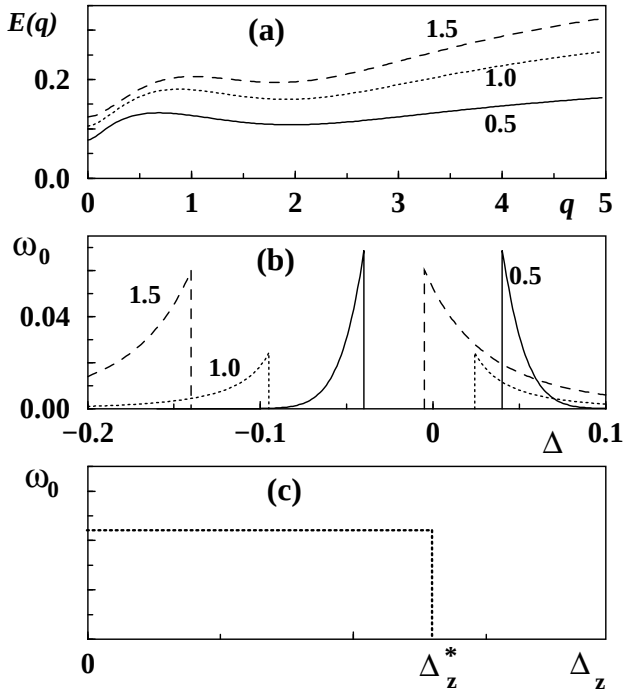

FIG. 3. (a) The excitation dispersion $E(q)$ (Eq. (10)) for $\delta\Delta = 0 = \Delta_z$. The numbers near the lines show the values of the parameter b, where b and q are in units of $1/l$, $E(q)$ is in units of ε_C. (b) The phonon absorption rate ω_0 as a function of Δ for $\Delta_z = 0$. Solid, dotted and dashed lines are for $b = 0.5$, 1.0 and 1.5, respectively. The data for $b = 1.0$ and 1.5 are multiplied by 10. The absorption rate ω_0 is in units of 10^{10} s^{-1}, Δ is in units of ε_C. (c) Phonon absorption rate ω_0 as a function of Δ_z for constant Δ. Here ω_0 and Δ_z are in arbitrary units. The critical Zeeman energy Δ_z^* corresponds to the $S_1 \rightarrow F$ or $S_2 \rightarrow F$ phase transitions.